"Classical" model of discrete QFT: Klein-Gordon and electromagnetic fields

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December 14, 2011

Abstract

The purpose of this paper is to propose a "classical" model which is relativistic with respect to a "superluminal" (but finite) speed c_s, and would result in emergent quantum field theory. This paper is exclusively devoted to the path integral representation of Klein-Gordon and Maxwell fields. The key challenge to overcome is the fact that a "path integral" is taken over an infinite-dimensional space, yet we are attempting to embed it in four dimensions for the purposes of making it ontologically meaningful. The key idea used to achieve the embedding is a discretization of infinite-dimensional space. Once the discretization is done our "infinite-dimensional" space will contain a finite number of elements, which means that "there is enough room" to embed it into four dimensions. The purpose of this paper is to specify the embedding and come up with a "classical" model that would produce a Feynman path integral in terms of the "inverse" of that embedding.

1. Introduction

The goal of this paper is to interpret quantum field theory as an *emergent* outcome of a deterministic *mechanical* process occurring on a sub-quantum scale. Doing so has two goals. First of all, it satisfies our classical intuition. Secondly, "converting" QFT into a "classical" theory would allow us to "couple" it to gravity by hand, without quantizing the latter. However, in this paper itself we will not try to tackle gravity (although some of the attempts in that direction can be found in Section 5 of [2]). Likewise, in this paper we will not attempt to tackle the theory of quantum measurement either (but, again, some of the attempts to do so can be found in Section 5 of [2]). The exclusive focus of this paper is to re-interpret the path integral formulation of QFT in flat space.

If we were to neglect the issues related to quantum measurement, then single particle quantum mechanics would look exactly like "classical" wave mechanics. While from the mathematical point of view the same is true for non-relativistic multiparticle systems, the situation looks very differently ontologically. Whereas we can "touch" a wave function in

ordinary space, we can no longer "touch" one in configuration space. The situation, of course, gets even more difficult when we switch from many-body non-relativistic quantum mechanics to second quantization. The goal of this paper is to propose a definition of probability amplitudes in ordinary $\mathbb{R}^3$ which makes sense regardless of how many particles we are dealing with, and regardless of whether or not they can be created or annihilated. The key idea used to achieve that is to discretize the quantum field theory in such a way that the informational content of the discretized theory becomes "smaller" than the informational content of $\mathbb{R}^3$. That makes it logically possible to embed the former within the latter.

In the single particle representation of quantum mechanics, a path integral can be thought of as a prescription for evolving a *general* wave function ψ, as opposed to a transition probability from one δ-function to another. If, at a time t_1, the wave function was $\psi_1(\vec{x})$, then at a time t_2 it will be $\psi_2(\vec{x})$ where the latter is computed according to

$$\psi_2(x_2) = \int_{\gamma(t_1)=x_1}^{\gamma(t_2)=x_2} d^3x_1\, \psi_1(x_1)\, e^{iS(\gamma)} \ . \tag{1}$$

In the path integral representation of quantum field theory for coupled Klein-Gordon and Maxwell fields, we are to replace $\vec{x}_1$ with (ϕ_1, A_1^μ) and $\vec{x}_2$ with (ϕ_2, A_2^μ), where ϕ_1, ϕ_2, A_1^μ and A_2^μ are functions with domain on $\mathbb{R}^3$. The trajectory $\gamma(t)$ will be replaced with $(\phi, A^\mu)(x^\mu)$, where ϕ and A^μ are viewed as functions over $\mathbb{R}^4$. The functions over $\mathbb{R}^4$ can be thought of as "time trajectories" over a set of functions over $\mathbb{R}^3$ according to

$$\phi(t,\vec{x}) = [\phi(t)](\vec{x}) \ ; \quad A^\mu(t,\vec{x}) = [A^\mu(t)](\vec{x}) \ . \tag{2}$$

Accordingly, we will continue to identify t with time, without replacing it with x^μ, which will assume a preferred time foliation. Thus, we introduce a wave function $\psi(\phi, A^\mu)$, which evolves according to

$$\psi_2(\phi_2, A_2^\mu; t_2) = \int_{(\phi,A^\mu)(t_1,\vec{x})=(\phi_1,A_1^\mu)(\vec{x})}^{(\phi,A^\mu)(t_2,\vec{x})=(\phi_2,A_2^\mu)(\vec{x})} [\mathcal{D}\psi][\mathcal{D}A]\psi_1(\phi_1, A_1^\mu; t_1) e^{iS(\phi,A^\mu)} \ , \tag{3}$$

where the domain of ϕ and A^μ is $\mathbb{R}^4$, while that of the "indexed" fields, ϕ_1, ϕ_2, A_1^μ and A_2^μ is $\mathbb{R}^3$. The fields with domain in $\mathbb{R}^4$ are "flowing parameters" while the fields with domain in $\mathbb{R}^3$ are "fixed" boundary conditions.

The goal of this paper is to represent $\psi(\phi, A^\mu)$ as a "physical" field living in $\mathbb{R}^3$ and evolving with time; or, equivalently, as a field living in $\mathbb{R}^4$. The key challenge is the fact that the set of elements (ϕ, A^μ) is $\mathbb{R}^\infty$ instead. Our trick is to propose an M-point discretization of $\mathbb{R}^\infty$ while leaving $\mathbb{R}^3$ and $\mathbb{R}^4$ continuous. This will allow us to embed a "discretized" $\mathbb{R}^\infty$ into a "continuous" $\mathbb{R}^3$, since the former has fewer elements than the latter. The discretization will be introduced through an embedding

$$\vec{x}\colon D \to \mathbb{R}^3 \ , \ D = \{1, \cdots, N\} \ , \tag{4}$$

together with the function $q\colon D \to \{1, \cdots, M\}$, where $N \gg M$ is a fixed integer. The function $q(k) = q_k$ will effectively break D into M sub-lattices

$$D = D_1 \cup \cdots \cup D_M \ , \ D_a = \{k \in D \mid q_k = a\} \ . \tag{5}$$

For any given k, q_k will be referred to as the *tuning parameter* of k (or, more briefly, the *tuning* of k). We will then define the single *fixed* field configurations $\phi\colon D \to \mathbb{R}$ and $A^\mu\colon D \to \mathbb{R}^4$, and denote their values as

$$\phi(k) = \phi_k \; ; \quad A^\mu(k) = A^\mu_k \; . \tag{6}$$

The "discretization" of (ϕ, A^μ) will be identified with the set of restrictions of the above two fields to D_q:

$$(\phi_{(q)}, A^\mu_{(q)})(k) = \begin{cases} (\phi_k, A^\mu_k) & \text{if } k \in D_q \\ \text{Undefined} & \text{otherwise.} \end{cases} \tag{7}$$

This implies that we can "switch" from $(\phi_{(a)}, A^\mu_{(a)})$ to $(\phi_{(b)}, A^\mu_{(b)})$ *without* changing the apriori-given (ϕ, A^μ). All we have to do is to change the domain we are "restricting" (ϕ, A^μ) to. We can now "embed" $\psi(\phi_{(q)}, A^\mu_{(q)}; t)$ into $\mathbb{R}^3$ according to

$$k \in D_q \Rightarrow \psi_k(t) = \psi(\phi_{(q)}, A^\mu_{(q)}; t) \; . \tag{8}$$

The time dependence of $\phi_k(t)$ and $A^\mu_k(t)$ should be contrasted with the time independence of ϕ_k and A^μ_k. This can be understood from the fact that the "single configuration" (ϕ, A^μ) corresponds to a "many-element discretization" $\{(\phi_{(q)}, A^\mu_{(q)}) \mid q = 1, \cdots, M\}$. As long as that discretization is "dense enough", the latter can be time independent, which implies time independence of the former. On the other hand, $\psi_k(t)$ corresponds to the set of probability amplitudes of each of $(\phi_{(q)}, A^\mu_{(q)})$ "taking place", which needs to be time dependent. Eq. (8) implies that

$$(q_i = q_j) \Rightarrow (\psi_i(t) = \psi_j(t)) \; . \tag{9}$$

The time dependence of the above equation implies that it can only hold in one "preferred" reference frame. The need for a "preferred frame" is logically parallel to various widely acknowledged reasons for imposing a "preferred frame" in realistic interpretations of quantum mechanics. As long as Eq. (9) holds, we can "invert" Eq. (8) according to

$$\psi(\phi_{(a)}, A^\mu_{(a)}) = \psi_k \; , \quad q_k = a \; . \tag{10}$$

Eq. (9) will guarantee that the above is independent of the choice of k, as long as k satisfies the condition on the right hand side. Thus, we can view ψ_k together with Eq. (9) as "more fundamental" and claim that $\psi(\phi_{(a)}, A^\mu_{(a)})$ is emergent through Eq. (10).

Let us now be a little more careful and break the situation into four parts: "kinematics of ψ_k", "dynamics of ψ_k", "kinematics of $\psi(\phi_{(q)}, A^\mu_{(q)})$" and "dynamics of $\psi(\phi_{(q)}, A^\mu_{(q)})$". Our goal is to claim that the first two items are fundamental, while the last two are emergent. What we have said so far implies that the "kinematics of $\psi(\phi_{(q)}, A^\mu_{(q)})$" is well defined as long as the "dynamics of $\psi(x)$" satisfies Eq. (9). This statement, however, does *not* tell us anything about the "dynamics of $\psi(\phi_{(q)}, A^\mu_{(q)})$". In order to obtain the latter, we have to provide a *further modification* of the kinematics of ψ_k. In other words, we need to say

$$\frac{d\psi_k}{dt} = F(\psi_k, \cdots) + G(\psi_k, \cdots) \; , \tag{11}$$

where the definition of $F(\cdots)$ will result in consistent kinematics of $\psi(\phi, A^\mu)$ while the definition of $G(\cdots)$ will lead to the dynamics. More specifically, the implication of $F(\psi_k, \cdots)$

should be Eq. (9), while the implication of $G(\psi_k, \cdots)$ should be Eq. (3). The goal of Sec. 2 is to satisfy ourselves that we can, indeed, have superluminal effects which are prerequisites to $F(\cdots)$. The goal of Sec. 3 is to come up with the dynamics for $F(\cdots)$, based on the said superluminal effects. The goal of Sec. 5 is to come up with the dynamics of $G(\cdots)$. Finally, the goal of Sec. 4 is to come up with an appropriate discretization of Lagrangian that Sec. 5 will aim to reproduce. The final outcome will be given in Eq. (84): its first line corresponds to $G(\cdots)$, and the second line corresponds to $F(\cdots)$.

2. Formation of lattice and relativity-related issues

One thing that makes the $F(\cdots)$ part a lot less trivial than it could have been is the fact that we would like to avoid non-locality, even though we admit the violation of relativity. In order for the theory we consider to be relativistic, it has to have only *one* speed of signal propagation, c_0. On the other hand, in order for a theory to be local, it can have n different speeds, $c_1, \cdots, c_n$, as long as n is a fixed finite number and each c_k is fixed as well. In our paper, we introduce two fundamental speeds, c_o and c_s ("o" stands for "ordinary" and "s" stands for "superluminal") satisfying

$$3 \times 10^8 \text{ m/s} \approx c_o \ll \frac{L}{\delta t} \ll c_s < \infty \,, \tag{12}$$

where L is the size of a box our universe is embedded into and δt is a very small time interval; both L and δt are finite. The inequality $c_o < c_s$ implies the violation of relativity; on the other hand, the inequality $c_s < \infty$ implies locality. The "non-local" nature of Eq. (9) is made possible by the fact that $L/\delta t \ll c_s$, which makes c_s "appear" infinite even if it isn't.

The above stands or fails on the assumption that L is finite. Our first impulse is to make L finite by claiming that our universe is compact. This, however, turns out to be unsatisfactory without a lot of extra care. After all, the compactness of the universe implies that the superluminal signals will repeatedly come back to the same point over and over again as they circle the universe. By conservation of energy we know that the effect of these signals will persist indefinitely in time. Yet, as long as the universe is anything other than a perfect sphere, the shape of the signals will become more and more chaotic as they bounce back and forth. This will imply that we will have a random noise with infintite lifetime. Since the "noise" consists of the signals emitted from the beginning of the universe up till recent past, it will dominate any kind of superluminal signals that might be emitted at present. In light of the fact that we are interested in the effects of presently-emitted superluminal signals, this is unsatisfactory.

One way to avoid this situation is to introduce a velocity-dependent "friction" to the dynamics of superluminal signals. In other words, the wave equation takes the form

$$g_s^{\mu\nu}\, \partial_\mu \sigma\, \partial_\nu \sigma + b\, \frac{\partial \sigma}{\partial t} = \text{source term} \,, \tag{13}$$

where b is what we are referring to as friction and g_s is a metric that enforces the speed c_s; thus, in flat space,

$$g_s = \text{diag}(c_s^2, -1, -1, -1) \,; \quad g_o = \text{diag}(c_o^2, -1, -1, -1) \,. \tag{14}$$

This means that the superluminal signals can attenuate in time. However, some important questions should still be asked. How much do they attenuate by the time they circle the universe? Are their "repeated effects" as they come back negligible? If yes, does it mean that they won't produce the desired effect on "far away" points during their "first run"? If not, do the repeated effects interfere with the "step by step" process of path integration we would like to introduce? In Ref. [2] some attempts were made to tackle these issues. However, it turns out that the extra machinery ended up being quite complicated, which leaves one wondering if there is a way to avoid the need for it.

In this paper we will try a different approach: we simply say that the universe is *not* compact. Thus, the signal "flies away" to infinity and never comes back to begin with. However, in light of the finiteness of the speed of the superluminal signals, we still have to demand that the lattice be embedded within a box of finite size L. In light of the non-compactness of the universe, we have to come up with a mechanism that would "make" the lattice finite. We will, therefore, propose that our lattice is "growing". This will logically explain its finite size by a finite age of a universe. Thus, by the time it "grows" more, it will become "too large" for superluminal signals to pass it within a very small time, and then quantum field theory will break down. But we assume that this will only happen in the far future.

We will set up our proposed growth model in the following way. Lattice points will be viewed as "charged particles" interacting through an electromagnetic-like potential V^μ (which has nothing to do with the "actual" electromagnetic field A^μ; it simply has a similar mathematical structure) that obeys

$$\partial_\mu(\partial^\mu V^\nu - \partial^\nu V^\mu) = \sum_{k=1}^{N} \int_{\gamma_k} \delta^4(x - \gamma_k(\tau))\, d\tau \ , \tag{15}$$

where there are N lattice points (numbered $k = 1, \cdots, N$) and $x^\mu = \gamma_k^\mu(\tau)$ is the trajectory of lattice point number k in spacetime. The worldline of a lattice point is determined by that vector potential; but it is quite different from what one would expect in electrodynamics. First of all, the lattice points are assumed to have the "same charge" (namely $+1$); secondly, the "same charge" interaction has attractive effects at large distances and repulsive effect at close distances, which would lead to the formation of a lattice with a "preferred" distance scale. This can be accomplished by imposing the following dynamical equation:

$$\frac{\partial^2 x^\mu}{\partial \tau^2} = a g_s^{\mu\nu} \partial_{\perp v;\mu}(V^\nu V_\nu - W^2)^2 \ ; \quad v^\mu = \frac{dx^\mu}{d\tau} \ , \tag{16}$$

where, for any given v^μ, the "parallel derivative" $\partial_{\parallel v;\mu}$ and the "orthogonal derivative" $\partial^\mu_{\perp v}$ are given by

$$\partial_{\parallel v;\mu} f = \frac{g_{s;\mu\rho} v^\rho v^\sigma \partial_\sigma f}{g_{s;\mu\nu} v^\mu v^\nu} \ ; \quad \partial_{\perp v;\mu} = \partial_\mu f - \frac{g_{s;\mu\rho} v^\rho v^\sigma \partial_\sigma f}{g_{s;\eta\chi} v^\eta v^\chi} \ . \tag{17}$$

The above definitions of "parallel" and "orthogonal" derivatives can be motivated by the observation that, in the (t, x, y, z)-convention,

$$v = (v, 0, 0, 0) \Rightarrow [\partial_{\parallel v} f = (\partial_0 f, 0, 0, 0, 0) \ ; \ \partial_{\perp v} f = (0, \partial_1 f, \partial_2 f, \partial_3 f)] \ . \tag{18}$$

The effect of the above potential is to attract any given particle j to the location

$$g_{s;\mu\nu}V^\mu(x_j)V^\nu(x_j) \approx W^2 \ . \tag{19}$$

If we impose initial conditions

$$x^0 = 0 \ \Rightarrow \ V^\mu(x) = 0 \ , \tag{20}$$

then $g_{s;\mu\nu}V^\mu(x_j)V^\nu(x_j) \approx W^2$ will be satisfied on a "preferred" distance scale. At the same time, when the particles are far away from each other, the attractive force is not terribly strong. Thus, they are moving around at their initial velocity and whenever they happen to pass by each other by accident, they get "stuck" to each other.

Now, if the number of points is finite, then the infinite size of the universe implies arbitrary large distance between any two points, which would imply zero probability of the formation of the lattice structure. On the other hand, if we assume that the number of points is infinite, then any given point will be subject to an infinitely large superluminal influence from far away. Take, for example, Eq. (15). This equation implies that the contribution towards V^μ from the distant points behaves like $1/r$. This means that the contribution from the points on the surface of radius r is of the order of r. Therefore, the integral over all r will produce infinity. In order to avoid this problem, we will have to introduce a damping parameter. The first impulse is to simply introduce a $\partial V^\mu/\partial t$ term, in a "preferred" time t. But, for aesthetic reasons, we would rather maintain c_s-based relativity (even though it would still violate the c_o-based one). Thus, instead of $\partial V^\mu/\partial t$, we will use $g_{\rho\sigma}V^\rho V^\mu \partial_\mu V^\sigma$, and Eq. (15) becomes

$$\partial_\mu(\partial^\mu V^\nu - \partial^\nu V^\mu) + \lambda\, g_{\rho\sigma}V^\rho V^\mu\, \partial_\mu V^\sigma = \sum_{k=1}^{N} \int_{\gamma_k} \delta^4(x - \gamma_k(\tau))\, d\tau \ . \tag{21}$$

The above depends on the assumption that V^μ is timelike, and has positive time component. This is a consequence of the initial conditions (20), together with the fact that $v^\mu = dx^\mu/d\tau$ is timelike with positive time component. We will introduce similar damping components towards any other wave equations we will be dealing with in the next sections:

$$g_s^{\alpha\beta}\partial_\alpha\partial_\beta\mu \mapsto g_s^{\alpha\beta}\partial_\alpha\partial_\beta\mu - \lambda\, V^\alpha\partial_\alpha\mu \ . \tag{22}$$

This will allow us to say that we have infinitely many points in the universe, but the influence from "far away" points is arbitrarily small. This will further allow us to assume that the average number of points per unit volume on a spacelike hypersurface is finite rather than infinitely small. However, we can assume that the average distance between points (which is likewise finite) is much larger than the one required for equilibrium, per Eq. (21). Thus, they still have to "run onto each other" in order to form a lattice. But this time the probability of this happening is non-zero. This further implies that, if we "wait long enough", the lattice will form with absolute certainty. In fact, infinitely many lattice structures will be forming; we claim that we are living in one of them.

Now, we claim that the entire universe that we live in is just one of these several lattice structures. The coefficient λ in Eq. (21) is so small that the size of the lattice structure we

are living in is not large enough for its effect to be felt. Thus, we will be using Eq. (15) as a close approximation throughout the rest of the paper. At the same time, however, the nearest point *outside* our lattice will be expected to be "far enough" for the λ-term to be significant. This apparent statistical contradiction can be accommodated by an assumption that the average *finite* distance between any *two* given points is "much larger" than the size of the entire lattice we are living in; and that is despite the fact that the latter includes billions of points! More precisely, if the number of points in the lattice we are living in is $N(t)$, and if the average density of points inside the lattice is $\rho_{\rm in}$ and outside is $\rho_{\rm out}$, then

$$\text{Our Time Period} \Rightarrow N(t)\,\rho_{\rm out} \ll \rho_{\rm in}\ . \tag{23}$$

The discrepancy between $\rho_{\rm out}$ and $\rho_{\rm in}$ is due to their very different origins. The density $\rho_{\rm out}$ is entirely based on the original distribution of points and is independent of their dynamics; while $\rho_{\rm in}$ is an "equilibrium density" determined entirely from the *dynamical* Eq. (21). Now, since the above drastic discrepancy is still finite, the lattice is still guaranteed to form if we wait long enough. Once the lattice has formed, it is being held together per either Eq. (15) or (21) (which closely approximate each other on this scale). At the same time, nothing "holds" any of the points outside the lattice. Thus, the nearest point *outside* the lattice is separated by an expected distance of the order of $1/\rho_{\rm out}^3$, which is several magnitudes larger than the size of the lattice. This will allow us to assume that λ is "large enough" for the total effect of *all* points outside the lattice to be negligible and, at the same time, λ is "small enough" for Eq. (15) to be a very close approximation to Eq. (21) *inside* the lattice.

It should be noticed that the spacing between lattice points might not be constant, due to the effects any given point experiences from "far away" points *in the same lattice*. In particular, closer to the edges of the lattice the spacing might end up being different than it is towards the center. Now, as was pointed out in Ref. [3], one can expect the ultraviolet cutoff to be inversely proportional to the spacing of lattice points. Thus, we might expect the ultraviolet cutoff to change as we move across the lattice; this would ultimately imply variations of the renormalized mass, charge, and so forth, assuming that the bare parameters are the same. However, we can claim that the part of the universe that is accessible to our observations is "very small" compared to the size of the lattice; thus, the variation of the ultraviolet cutoff is negligible within the region accessible to our observations.

This, however, assumes that the spacing between lattice points approximates a continuous function. This, too, can be questioned. It is possible that small-scale interactions would lead to non-trivial distance variations even on few-point scales. Nevertheless, one can still expect that the pattern of discontinuities would form some kind of repeated structure. This structure might average to something continuous. In other words, we might have three "very small" space scales δ_1, δ_2 and δ_3 which satisfy $\delta_1 \ll \delta_2 \ll \delta_3$. We can have discontinuities on the scale of δ_1 which would average to something continuous on the scale of δ_2. A function that is "continuous" on the scale δ_2 will, in fact, vary on the scale of δ_3; yet, δ_2 is too small for variation to occur, which makes the function nearly-constant on that scale. We can then assume that $\delta_3 \ll L$ and *despite that*, δ_2 is the scale of the observable universe. This will allow a variation of the ultraviolet cutoff throughout lattice (whose scale is L) while at the same time that variation would be "too far away" for us to see.

In this framework it is possible to argue that the theory of relativity *is* satisfied, after all. However, the version of relativity that is satisfied is c_s-based rather than c_o-based. On the other hand, c_o is a parameter that strictly applies to the lattice that has already been formed. Thus, the lattice identifies the "preferred frame" in much the same way as the water identifies a "preferred frame" for the propagation of water waves; and just like the water waves move slower than the speed of light c_o, at the same time the photons move much slower than the speed c_s. Furthermore, since different lattices can form at the same time, they will be moving relative to each other. Thus, each lattice will carry its own "preferred frame"; just like the geological processes of the Earth are based on Earth's preferred frame while similar processes in the Moon are based on the frame of the Moon. This illustrates that in reality neither lattice is a true preferred frame. Instead, we have a strict relativity, based on c_s.

It should be pointed out, however, that the belief in c_s-based relativity is logically independent of any of the experiments we have performed that taught us relativity. After all, it is perfectly conceivable that *despite* c_s-based relativity, the processes we see within a given lattice demonstrate the violation of relativity (just like the processes we see in the ocean do). The reason relativity is respected is that we have "cleverly designed" each lattice in such a way that it is (this has been done in Section 4). For example, it could have been possible to have electricity without magnetism, which would have demonstrated violation of relativity; but we "cleverly" introduced magnetism in order to "hide" this. Our ability to "design" a lattice in such a way that it respects c_o-based relativity is independent of the validity of the c_s-based one, as evident from the difference between the values of c_o and c_s. Our insistence on the validity of c_s-based relativity is only aesthetic and is not backed up by any experimental evidence.

3. Definition of probability amplitude

In the previous section we have proposed a model of formation and dynamical growth of a lattice. We have also argued that, at this point in time, the lattice is "small enough" for superluminal signals to cross it within a negligible time period. Let us now proceed to the more central part of the paper. We will now use the above superluminal signals in order to make sure that Eq. (9) is satisfied which, per the argument made in Introduction, will imply the existence of well-defined $\psi(\phi_{(q)}, A^{\mu}_{(q)})$.

We propose the following model. First, we number the dots. Thus, we have dots 1 through N, with the k-th dot located at $\vec{x}_k$ (which first moves according to Eq. (16) and then becomes stationary at its equilibrium position, which is assumed to have been reached by now). We attach to each dot number k a "clock" in the form of an oscillator $e_k(\tau)$, which evolves in time according to

$$\frac{d^2 e_k}{d\tau^2} = -\nu^2 e_k \ , \tag{24}$$

where $d\tau$ is defined in terms of the metric g_s instead of g_o:

$$d\tau^2 = g_{s;\mu\nu}\, dx^{\mu} dx^{\nu} \ . \tag{25}$$

Thus, these parameters "oscillate" with the same "frequency" ν across different lattice points, although the amplitudes and phases of their oscillations differ:

$$e_k = E_k \cos(\nu\tau + \alpha_k) \; . \tag{26}$$

Now, we also postulate the existence of a very small, but finite, constant $\chi \ll 1$. Whenever the phase of the i-th oscillator "crosses" the interval $[-\chi, \chi]$ (which means that a "clock" shows a particular time), it sends a pulse with "superluminal" (but finite) speed $c_s \gg c_o$, as described in the previous two sections. In light of the fact that χ is small, that pulse has a very short duration. During the passage of that pulse, all of the dots j that have the same "tuning parameter" change their respective values of ψ_j in such a way that they "match" the value of ψ_i, while ψ_i itself remains unchanged. Thus, the only time when the "correlation of ψ_j" among dots with the same tuning did not hold is the very beginning of the universe, when none of the dots of a given tuning had had an opportunity to send their pulses. After the very first pulse, the value of ψ_j among same-tuning dots becomes the same; and it coincides with the initial value of ψ_j at the dot that was the *first* to send a pulse.

Now, in order for the pulses to accomplish the desired effect, they have to carry the information about the internal parameters of the dot that sent these pulses. For example, in order for dot j to "copy" the value of ψ_i, the latter has to be somehow "carried" by the pulse, since locality prevents dot j from having direct access to dot i. Furthermore, since we only want to do the "copying" if $q_j = q_i$, dot number j needs to "know" the value of q_i. Thus, the pulse needs to "carry" the latter as well. Finally, as we will soon see, in order to "find out" either of these two things, we need to know the distance to the dot. Thus, we formally introduce three "image fields" $I_q(x)$, $I^\mu_x(x)$ and $I_\psi(x)$ whose values approximate q_i, $x^\mu_{\perp v} - x^\mu_{i\perp v}$ and ψ_i respectively:

$$(I_q, I^\mu_x, I_\psi)(x^\mu) \approx \begin{cases} (q_j, x^\mu_{\perp v} - x^\mu_{i\perp v}, \psi_j) & \text{if dot } j \text{ emitted the signal reaching } \vec{x} \text{ at time } t \\ (0,0,0) & \text{otherwise.} \end{cases} \tag{27}$$

In light of the "classical" nature of the desired theory, Eq. (27) cannot be simply postulated. Instead, we want to come up with a set of differential equations that produces the latter. In fact, we would like the definitions of I_q, I^μ_x and I_ψ per the sought-after differential equations to be exact, while Eq. (27) will be an emergent approximation. As we mentioned previously, the "source" of a signal is a particle whose oscillator crosses the region $[-\chi, \chi]$. Thus, the source term needs to include a "conditional" function. For this purpose, we will define the "truth value" of a statement as follows:

$$T(\text{true}) = 1 \; ; \quad T(\text{false}) = 0 \; . \tag{28}$$

In order to accommodate a reader who "for philosophical reasons" wants everything to be differentiable whenever possible, we will introduce a differentiable approximation to the above given "truth value":

$$T_\epsilon(x \in [a,b]) = \Big(\frac{1}{2} + \frac{2}{\pi}\tan^{-1}\frac{x-a}{\epsilon}\Big)\Big(\frac{1}{2} + \frac{2}{\pi}\tan^{-1}\frac{b-x}{\epsilon}\Big) \; . \tag{29}$$

The wave operator for the wave propagating at the speed c_s is $g_s^{\mu\nu}\,\partial_\mu\partial_\nu$. The signals propagating under this operator would decrease their strength over distance, contrary to Equation 27. Therefore, we will introduce two sets of fields: μ-s and I-s. The μ-fields will attenuate with the distance, while I-fields will not. The analysis of μ-fields will make it possible to deduce the distance to source (based on "local" information alone) and, therefore, "manufacture" non-attenuating expressions, which will be identified with I-s.

Let us now write it more explicitly. The μ-fields will behave according to the wave equation:

$$g_s^{\alpha\beta}\partial_\alpha\partial_\beta\mu_q - \lambda_{\mu_q} g_s^{\alpha\beta} V_\alpha \partial_\beta\mu_q + m_{\mu_q}^2\mu_q = \sum_{j=1}^{N}\Big(q_j\int_{\gamma_j}\delta^4(x-\gamma_j(\tau))\,T_\epsilon\Big(\frac{1}{e_j}\frac{de_j}{d\tau}\in[-\chi,\chi]\Big)\Big)$$

$$g_s^{\alpha\beta}\partial_\alpha\partial_\beta\mu_\psi - \lambda_{\mu_\psi} g_s^{\alpha\beta} V_\alpha \partial_\beta\mu_\psi + m_{\mu_q}^2\mu_\psi = \sum_{j=1}^{N}\Big(\psi_j\int_{\gamma_j}\delta^4(x-\gamma_j(\tau))\,T_\epsilon\Big(\frac{1}{e_j}\frac{de_j}{d\tau}\in[-\chi,\chi]\Big)\Big)\,,$$

where the purpose of the Ts is to make sure that the signals are emitted in pulses (and the duration of the pulses is limited by the time period where the "conditions" of the Ts are satisfied). As before, the purpose of the λ-term is to provide a very small friction coefficient that would be unnoticeable within the same lattice but prevent communication across different lattices. And, again as was mentioned earlier, the V_α in the λ-term is basically a c_s-covariant replacement of δ_α^0 which is guaranteed to be timelike and positive due to the initial conditions (20).

If we notice that the signal has a finite duration and we assume that it has "started" some time ago and will continue on for some more time, we can use the spherical symmetry in $\mathbb{R}^3$ to say that μ_q is inversely proportional to the projected distance on a spacelike hypersurface, $|x_{\perp V} - \gamma_{\perp V}(\tau)|$, to an approximation of the order of a function of the "friction term" λ_μ. Therefore, it is easy to check that the desired conditions for I_x^α, I_ψ and I_q are approximately met during the majority of the time of a pulse if we define these according to

$$I_x^\alpha = \frac{e^{-\kappa/\mu_q}\mu_q g_s^{\alpha\beta}\partial_{\perp v;\beta}\mu_q}{g_s^{\gamma\delta}\partial_{\perp v;\gamma}\mu_q\partial_{\perp v;\delta}\mu_q}\,;\quad I_\psi = \frac{e^{-\kappa/\mu_q}\mu_q\mu_\psi}{\sqrt{g_s^{\alpha\beta}\partial_\alpha\mu_q\partial_\beta\mu_q}}\,;\quad I_q = \frac{e^{-\kappa/\mu_q}\mu_q^2}{\sqrt{g_s^{\alpha\beta}\partial_\alpha\mu_q\partial_\beta\mu_q}}\,. \tag{30}$$

Here, κ is very small; thus, throughout the duration of the signal, $e^{-\kappa/\mu_q}\approx 1$ which is what we need to assume in order for the first line on the right-hand side of Eq. (27) to be satisfied. On the other hand, whenever μ_q is very small, we get $e^{-\kappa/\mu_q}\approx 0$. In fact, $e^{-\kappa/\mu_q}$ goes to zero much faster than anything else that might go to infinity, which is why we obtain $I_x^\alpha \approx I_\psi \approx I_q \approx 0$, which means that the second line of Eq. (27) is satisfied as well.

So now we finally found a way of "encoding" the values of q_j and $\psi_j$within the "fields" $I_q(\vec{x},t)$ and $I_\psi(\vec{x},t)$. This means that we can "rewrite" our statement about "copying of q_j" in terms of these latter fields and, therefore, avoid non-locality. Let us do it explicitly. As we recall, the original "copying" law takes the following form:

$$\text{Passing signal from } i \Longrightarrow \psi_j(\tau+\chi)\approx\begin{cases}\psi_i(\tau-\chi) & \text{if } q_i = q_j\\ \psi_j(\tau-\chi) & \text{otherwise.}\end{cases} \tag{31}$$

Since we are "sitting" on a dot j, it is "fine" to make a reference to q_j; but we can *not* make a reference to q_i. Thus, we have to replace q_i with $I_q(\vec{x},t)$. Since $I_q(\vec{x},t)$ coincides with q_i independently of $\vec{x}$, we could have picked any $\vec{x}$ we like. But, for our purposes, we want to be "local" with respect to dot j. Thus, we pick $\vec{x}=\vec{x}_j$, which gives us

$$\text{Passing signal} \Longrightarrow \psi_j(\tau+\chi) \approx \begin{cases} I_\psi(x_j^\mu(\tau-\chi)) & \text{if } q_j \approx I_q(\vec{x}_j,t) \\ \psi_j(t-\chi) & \text{otherwise.} \end{cases} \tag{32}$$

The expressions $I_\psi(x^\mu(\tau-\chi))$ and $\psi_j(t-\chi)$ should not be confused with each other. The former describes a field that "fills up" the entire space which we *happened to evaluate* at $\vec{x}=\vec{x}_j(\tau)$ while the latter describes an internal degree of freedom of a dot j and does not exist outside of that dot. In other words, the former field is defined in the "space between lattice points" while the latter is only defined *at* the lattice points. Clearly, the two do not coincide. After all, $I_\psi(x_j^\mu(\tau-\chi))$ approximates ψ_i as opposed to ψ_j.

Now, the above expression is designed for the duration of the passage of a signal. But, strictly speaking, the difference between "signal" and "no signal" is quantitative rather than qualitative. The relevant fields are defined at all times (whether a "signal" is emitted or not); they are just "much stronger" during the time signal passes. Therefore, we should generalize the above expression for all times. It turns out that we can naively rewrite the above expression and simply remove the condition that a "signal is passing". If no signal passes, the above expression will "on its own" imply that ψ_j will not change:

$$\text{No signal } \Rightarrow\ I_q(x_j^\mu(\tau)) \approx 0\ \Rightarrow\ q_j \not\approx I_q(x_j^\mu(\tau))\ \Rightarrow \psi_j(\tau+\chi) \approx \psi_j(\tau-\chi)\ . \tag{33}$$

Thus, we rewrite the expression while replacing "passing signal" with "arbitrary time":

$$\text{Arbitrary time} \Longrightarrow \psi_j(\tau+\chi) \approx \begin{cases} \psi(\vec{x}_j,\tau-\chi) & \text{if } q_j \approx I_q(x_j^\mu(\tau)) \\ \psi_j(\tau-\chi) & \text{otherwise.} \end{cases} \tag{34}$$

We would now like to write down differential equations that would guarantee the above. It can be readily seen that, for "very large" parameters K and A, a differential equation of the form

$$\frac{d\psi_j}{d\tau} \approx B(I_\psi(x_j^\mu(\tau)) - \psi_j(\tau))\, e^{A(I_\psi(x_j^\mu(\tau))-\psi_j(t))^2} T(q_j \approx I_q(\vec{x}_j,t)) \tag{35}$$

accomplishes the stated goal. It should be noticed that I_q is nearly-integer at all times. When no signals pass, I_q approximates 0, and 0 is an integer. When a signal passes, I_q approximates the value of the "tuning" of the dot that has emitted the pulse, which is also an integer. Whenever the "integer" value of I_q happens to be zero, we *know* that there are no pulses. After all, all of the dots have (integer) values of tuning, which are greater or equal to 1. Likewise, whenever there are no pulses, the near-zero value of I_q is guaranteed to "mismatch" the value of a "tuning" a given dot i carries. Thus all cases are well taken care of. Now, in light of the fact that I_q is near-integer, we can rewrite the above "approximation" as

$$T(q_j \approx I_q(x_j^\mu(\tau))) = T_\epsilon\Big(I_q(x_j^\mu(\tau))\Big| q_j - \frac{1}{2} < I_q(x_j^\mu(\tau)) < q_j + \frac{1}{2}\Big)\,. \tag{36}$$

We now substitute that approximation and write down the following equation:

$$\frac{d\psi_j}{dt} = B(I_\psi(x_j^\mu(\tau)) - \psi_j(\tau))e^{A(\psi(\vec{x}_j,t)-\psi_j(t))^2} T_\epsilon\Big(I_q(\vec{x}_j,t)\Big| q_j - \frac{1}{2} < I_q(\vec{x}_j,t) < q_j + \frac{1}{2}\Big)\,. \tag{37}$$

It should be noticed that the above equation is "exact", despite the use of a *finite* constant ϵ. After all, we can "postulate" whatever differential equation we like, and whatever we postulate automatically becomes "exact". Thus, there is one "preferred" small *but finite* value of ϵ for which the above equation is exact, while for all of the other values of ϵ, *including the limit of* $\epsilon \to 0$, the above is only an approximation! In particular, the "desired" results we were proposing earlier only hold "approximately"; but, due to the right choice of the "exact" equation we have just proposed, that approximation is close enough.

4. The desired discrete theory

So far we have established that we have a lattice (which was formed according to the dynamics described in Sec. 2) and we have also established that we have a consistent definition of probability amplitudes on that lattice, according to Sec. 1 and Sec. 3. This was done by means of splitting the lattice into several sublattices, where the "number" of each sublattice is a common tuning parameter of any given point that is part of that sublattice. Finally, in Sec. 1 we have shown a link between the probability amplitudes we "encode" and a Feynman path integral. However, we have not specified the Lagrangian we are taking the path integral with. In fact, the emergent outcome of the dynamics in Sec. 3 is a consistent probability amplitude in the case of the Lagrangian being zero! We would now like to do the next step: specify the Lagrangian and include it in the path integral of Sec. 1. This means that we have to modify the dynamics of Sec. 3 in a way that logically reflects the Lagrangian in question. Strictly speaking, Sec. 3 in its current form is the $F(\cdots)$ of Eq. (11), and we would now like to do $G(\cdots)$.

In light of the fact that the theory is discrete, the first step is to discretize the Lagrangian of quantum field theory. Only the *next* step will be to come up with modifications of dynamics that will produce that Lagrangian as an emergent phenomenon. Therefore, in this section we will put aside our wish to be "classical" and we will focus exclusively on the "quantum mechanical" problem of discretizing QFT. Then in the next section we will go back and ask ourselves what "classical" theory would lead to the discrete QFT we are now introducing. Since this section is a "quantum mechanics aside", so to speak, we will forget any of the key constructs we were introducing, including different tuning parameters and so forth, as far as this particular section is concerned.

Since we "secretely know" that our lattice has formed due to rather complicated interactions, we can not assume that the lattice is cubic. Instead, we will assume a Poisson distribution in $\mathbb{R}^3$ of points, each of which forms a worldline in $\mathbb{R}^4$. Since we also "secretely know" that the points inside the lattice reached equilibrium positions, we will postulate them to be straight lines pointing in the same direction. In terms of the model proposed in previous sections, the "common directions" might be different for different lattices. However, since the interaction between different lattices is very small, we are not concerned about any other lattice. Thus, we can pick a time axis to coincide with the common velocity of the points within the lattice that we are living in. This means that we have a Poisson distribution in $\mathbb{R}^3$ of worldlines parallel to the t-axis in $\mathbb{R}^4$. Once again, however, in this section we are not concerned about any "mechanisms" or "models" we are "secretly believing" in. Thus, as far

as this section is concerned, the Poisson distribution of lines parallel to the t-axis is a simple mathematical postulate. Then in the next section we will go back and re-introduce physics.

Now, since we are not assuming a cubic lattice, there is no "obvious" way of "taking derivatives". After all, representing derivatives in terms of finite differences requires using three preferred neighbors, one along each of the three axes. This would, again, violate rotational symmetry, which is precisely what we don't want. We would like, therefore, to define derivatives in a "statistical" way taking into account several points. This goal can be accomplished if we express derivatives in an integral form. In particular, we introduce a *Lagrangian generator* $\mathcal{K}(x^\mu, y^\mu, z^\mu; \text{fields})$, then define a *Lagrangian density* $\mathcal{L}(\vec{z}, t)$ as

$$\mathcal{L}(\vec{z}, t; \text{fields}) = \int d^d x \, d^d y \, \mathcal{K}((\vec{x}, t - \delta t), (\vec{y}, t - \delta t), (\vec{z}, t); \text{fields}) \, , \tag{38}$$

and then define the action according to

$$S(\text{fields}) = \int d^d z \, dt \, \mathcal{L}(\vec{z}, t; \text{fields}) \, . \tag{39}$$

In fact, there is a "mechanical" way of "reading off" the integral expression by "looking" at the derivatives, without any extra work! Let us first define the following functions:

$$(\Gamma_{0\alpha}\phi)(x^\mu, z^\mu) = \frac{\alpha^{1/2}}{(2\pi)^{1/2}\delta t} \, e^{-\alpha \, |\vec{z} - \vec{x}|^2/2} \, (\phi(z^\mu) - \phi(x^\mu)) \tag{40}$$

$$(\Gamma_{p\alpha}\phi)(\vec{x}, \vec{z}) = \frac{\alpha^{(d+2)/2}}{(2\pi)^{d/2}} \, e^{-\alpha \, |\vec{z} - \vec{x}|^2/2} \, (z^p - x^p)(\phi(z^\mu) - \phi(x^\mu)) \tag{41}$$

$$I_\alpha(x^\mu, z^\mu) = \Big(\frac{\alpha}{2\pi}\Big)^{d/2} e^{-\alpha \, |\vec{z} - \vec{x}|^2/2} \, , \tag{42}$$

where, for any given four-vector v^μ, we use a "three-vector" notation $\vec{v}$ whenever we "drop" the time index 0:

$$\mu \in \{0, \cdots, d\} \Rightarrow \vec{v} = (v^1, \cdots, v^d) \, . \tag{43}$$

Furthermore, on the place of ϕ we can substitute a vector field, such as A^μ. While there is a fundamental difference between $\Gamma_{0\alpha}$ and $\Gamma_{p\alpha}$, we do *not* have any fundamental difference between A^0 and A^p. After all, the index that A^μ carries simply "comes for a ride". Thus, it can retain its Lorentzian symmetry. Thus, we need *two* definitions (for $\Gamma_{0\alpha}A^\mu$ and $\Gamma_{p\alpha}A^\mu$) *as opposed to* four definitions (for $\Gamma_{0\alpha}A^0$, $\Gamma_{0\alpha}A^p$, $\Gamma_{p\alpha}A^0$ and $\Gamma_{p\alpha}A^q$). Our "two" definitions are a carbon copy of definitions for $\Gamma_{0\alpha}\phi$ and $\Gamma_{p\alpha}\phi$ with ϕ being replaced by A^μ:

$$(\Gamma_{0\alpha}A^\mu)(x^\mu, z^\mu) = \frac{\alpha^{1/2}}{(2\pi)^{1/2}\delta t} e^{-\frac{\alpha}{2}|\vec{x}_i - \vec{x}_k|^2} (A^\mu(i) - A^\mu(k)) \tag{44}$$

$$(\Gamma_{p\alpha}A^\mu)(k, i) = \frac{\alpha^{(d+2)/2}}{(2\pi)^{d/2}} e^{-\frac{\alpha}{2}|\vec{x}_i - \vec{x}_k|^2} (x_i^p - x_k^p)(A^\mu(i) - A^\mu(k)) \, . \tag{45}$$

Let us now try a Lagrangian Generator of the form

$$\mathcal{K}(x^\mu, y^\mu, z^\mu; \phi) = [(\Gamma_{0\alpha}\phi)(x^\mu, z^\mu)][(\Gamma_{p\alpha}\phi)(y^\mu, z^\mu)] \, , \tag{46}$$

where we have violated the rotational symmetry by selecting a "preferred" coordinate $p \in \{1, \cdots, d\}$. By substituting the expressions for Γs, we obtain

$$\mathcal{K}(x^\mu, y^\mu, z^\mu; \phi) = \Big[\frac{\alpha^{1/2}}{(2\pi)^{1/2}\delta t}\, e^{-\alpha\,|\vec{z}-\vec{x}|^2/2}\,(\phi(z^\mu) - \phi(x^\mu))\Big]\times$$

$$\times\,\Big[\frac{\alpha^{(d+2)/2}}{(2\pi)^{d/2}}\, e^{-\alpha\,|\vec{z}-\vec{y}|^2/2}\,(z^k - y^k)(\phi(z^\mu) - \phi(y^\mu))\Big]\,. \tag{47}$$

Now, by inspecting Equation 38, we see that the ingredients inside of the Lagrangian generator are "taken" at a different time. In particular, $x^0 = y^0 = z^0 - \delta t$. Therefore, we have to make the following substitutions:

$$\begin{gathered}\phi(x^\mu) \to f(\vec{x}, t) - \delta t \frac{\partial f}{\partial t}\Big|_{\vec{x}} \\ \phi(y^\mu) \to f(\vec{y}, t) - \delta t \frac{\partial f}{\partial t}\Big|_{\vec{y}} \\ \phi(z^\mu) \to f(\vec{z}, t)\,.\end{gathered} \tag{48}$$

Thus, the Equation 38 together with the Equation 56 as well as the substitutions above produces

$$\mathcal{L} = \int d^dx d^dy \Big\{\Big[\frac{\alpha^{1/2}}{(2\pi)^{1/2}\delta t} e^{-\frac{\alpha}{2}|\vec{z}-\vec{x}|^2}\Big(f(\vec{z}, t) - \Big(f(\vec{x}, t) - \delta t\,\frac{\partial f}{\partial t}\Big|_{\vec{x}}\Big)\Big)\Big]\times \tag{49}$$

$$\times\,\Big[\frac{\alpha^{(d+2)/2}}{(2\pi)^{d/2}} e^{-\frac{\alpha}{2}|\vec{y}-\vec{z}|^2}(y^p - z^p)\Big(f(\vec{z}, t) - \Big(f(\vec{y}, t) - \delta t\,\frac{\partial f}{\partial t}\Big|_{\vec{y}}\Big)\Big)\Big]\Big\}\,, \tag{50}$$

Now, if $\alpha \gg 1$, then we can assume that the only values of $\vec{x}$ and $\vec{y}$ that have non-negligible contribution to the integral are the ones that are very close to $\vec{z}$. Therefore, if we assume that f is well behaved, we can assume that f is linear in the region where its contribution is non-negligible:

$$f(\vec{x}, t) \approx f(\vec{z}, t) + (\vec{x} - \vec{z}) \cdot \vec{\nabla} f\,. \tag{51}$$

We can therefore rewrite the above Lagrangian as

$$\mathcal{L} = \int d^dx d^dy \Big\{\Big[\frac{\alpha^{1/2}}{(2\pi)^{1/2}\delta t} e^{-\frac{\alpha}{2}|\vec{z}-\vec{x}|^2}\Big(\delta t \frac{\partial f}{\partial t}\Big|_{\vec{x}} + (\vec{z} - \vec{x}) \cdot \vec{\nabla} f\Big)\Big]\times \tag{52}$$

$$\times\,\Big[\frac{\alpha^{(d+2)/2}}{(2\pi)^{d/2}} e^{-\frac{\alpha}{2}|\vec{y}-\vec{z}|^2}(z^p - y^p)(\vec{z} - \vec{y}) \cdot \vec{\nabla} f)\Big]\Big\}\,. \tag{53}$$

Now, the term $(\vec{x}-\vec{z})\cdot\vec{\nabla} f$ is odd with respect to $\vec{x}-\vec{z}$, and, therefore, drops out of integration. On the other hand, $(y^p - z^p)(\vec{y} - \vec{z}) \cdot \vec{\nabla}\phi$ produces terms of the form $(y^p - z^p)(y^q - z^q)\partial_q\phi$. The $p \neq q$ terms are, likewise, odd with respect to $\vec{y} - \vec{z}$ and, therefore, also drop out. At the same time, $p = q$ terms are even and, therefore, are left. These terms simplify as $(y^p - z^p)^2\partial_p\phi$. Therefore, our new expression becomes

$$\mathcal{L} = \int d^dx d^dy \Big\{\Big[\frac{\alpha^{1/2}}{(2\pi)^{1/2}\delta t} e^{-\frac{\alpha}{2}|\vec{z}-\vec{x}|^2} \times \delta t \frac{\partial f}{\partial t}\Big|_{\vec{x}}\Big] \times \Big[\frac{\alpha^{(d+2)/2}}{(2\pi)^{d/2}} e^{-\frac{\alpha}{2}|\vec{y}-\vec{z}|^2}(y^p - z^p)^2\partial_p f)\Big]\Big\}, \tag{54}$$

where it should be understood that $p \in \{1, \cdots d\}$ is a *fixed* integer, and *there is no* Einstein's summation convention in the above expression. By performing a simple, but rather routine, computation involving separation of variables, the above expression reduces to

$$\mathcal{L} = \frac{\partial f}{\partial t} \partial_p f \tag{55}$$

This is structurally very similar to the Lagrangian generator we started out with, which is given in the Equation 46,

$$\mathcal{K}(x^\mu, y^\mu, z^\mu; \phi) = [(\Gamma_{0\alpha}\phi)(x^\mu, z^\mu)][(\Gamma_{p\alpha}\phi)(y^\mu, z^\mu)] \; . \tag{56}$$

It can be checked that this similarity extends to other situations as well, which makes it very easy to "read off" the expression for $\mathcal{K}(x^\mu, y^\mu, z^\mu; \phi, A^\mu)$ if we are given an expression for Lagrangian density in sought-after continuum-based theory.

There is, however, one subtlety which leads to infinite overcountings. For example, suppose we want to "produce" $\mathcal{L} = \partial_p \phi$. Naively, we can "read off" the Lagrangian generator $\mathcal{K}(x^\mu, y^\mu, z^\mu) = (\Gamma_{p\alpha}\phi)(x^\mu, z^\mu)$. This means that the integrand will be $\vec{y}$-independent. *However*, we *still* have to formally take an integral over $\vec{y}$. This means that we will be taking integral over a constant, thus producing infinity. This is where $I_\alpha(y^\mu, z^\mu)$ comes along (which, by the way, is the very purpose for which it was introduced). We will incorporate $\vec{y}$-dependence by multiplying the Lagrangian generator by $I_\alpha(y^\mu, z^\mu)$:

$$\mathcal{L} = \partial_p f \iff \mathcal{K}(x^\mu, y^\mu, z^\mu) = [(\Gamma_{p\alpha} f)(x^\mu, z^\mu)][I_\alpha(y^\mu, z^\mu)] \; . \tag{57}$$

It can be easily checked that the function over y produced by $I_\alpha(k, b_k)$ will integrate to 1 and, therefore, will not affect the result. Likewise, if we don't have any derivatives at all, we will simply put two I_α-coefficients: one to "take care" of $\vec{x}$-dependence, and the other one to "take care" of $\vec{y}$-dependence. For example, the "mass term" $m^2\phi^2$ corresponds to "Lagrangian generator" according to

$$\mathcal{L} = m^2 \phi^* \phi \iff \mathcal{K}(x^\mu, y^\mu, z^\mu; \phi) = m^2 \big[\phi^*(k)\phi(k)\big] \big[I_\alpha(x^\mu, z^\mu)\big] \big[I_\alpha(y^\mu, z^\mu)\big] \; . \tag{58}$$

We are finally ready to write down the Lagrangian generator for the complete Lagrangians. Consider a charged scalar field interacting with electromagnetic field. The Lagrangian for the interaction is

$$\mathcal{L} = m^2\phi^*\phi + \mathcal{D}^\mu\phi^* \mathcal{D}_\mu\phi + F^{\mu\nu}F_{\mu\nu} \; , \; \mathcal{D}^\mu\phi = \partial^\mu\phi + ieA^\mu\phi \; ; \; \mathcal{D}^\mu\phi^* = \partial^\mu\phi^* - ieA^\mu\phi^* \; . \tag{59}$$

Upon some simple algebra, the above expression becomes

$$\mathcal{L} = F^{\mu\nu}F_{\mu\nu} + \partial^\mu\phi^*\partial_\mu\phi + e^2 A^\mu A_\mu \phi^*\phi + ieA^\mu(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) + m^2\phi^2 \; . \tag{60}$$

By remembering the expression for the current,

$$j^\mu = ieA^\mu(\phi\partial^\mu\phi^* - \phi^*\partial^\mu\phi) \; , \tag{61}$$

it is easy to see that the ie term is the interaction between the "current" and the photon. In fact, the only "surprising" term is a four-vortex $A^\mu A_\mu \phi^* \phi$. Upon further thought it can be realized that this term is not that surprising either: while such four-vortex is "forbidden" for spin 1/2 fermions (since the latter has dimension 3/2), it is "allowed" for spin 0 boson (which has dimension 1). For our purposes, we will rewrite the Lagrangian density as

$$\begin{aligned} \mathcal{L} = (g^{\rho\mu}\partial_\rho A^\nu - g^{\rho\nu}\partial_\rho A^\mu)(g_{\nu\rho}\partial_\mu A^\rho - g_{\mu\rho}\partial_\nu A^\rho) + \\ + \ e^2 g_{\mu\nu} A^\mu A^\nu \phi^* \phi + ieA^\mu(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) + g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + m^2\phi^2 \ . \end{aligned} \tag{62}$$

As one notices, in the above expression A^μ always come with the upper indexes and ∂_μ always comes with the lower indexes. The "lower index" of ∂_μ is crucial since this is what $\Gamma_{\mu\alpha}$ is designed to approximate. On the other hand, the upper index of A^μ is mostly aesthetic. We are now ready to "read off" the Lagrangian generator:

$$\begin{aligned} \mathcal{K}(x^\mu, y^\mu, z^\mu) = (g^{\rho\mu}(z^\mu)(\Gamma_{\rho\alpha}A^\nu)(x^\mu, z^\mu) - g^{\rho\nu}(z^\mu)(\Gamma_{\rho\alpha}A^\mu)(x^\mu, z^\mu)) \times \\ \times (g_{\nu\rho}(z^\mu)(\Gamma_{\mu\alpha}A^\rho)(y^\mu, z^\mu) - g_{\mu\rho}(k)(\Gamma_{\nu\alpha}A^\rho)(y^\mu, z^\mu)) + \\ + \ e^2 g_{\mu\nu}(z^\mu)A^\mu(z^\mu)A^\nu(z^\mu)\phi^*(z^\mu)\phi(z^\mu)I_\alpha(x^\mu, z^\mu)I_\alpha(y^\mu, z^\mu) + \\ + ieA^\mu(z^\mu)(\phi(z^\mu)(\Gamma_{\mu\alpha}\phi^*)(x^\mu, z^\mu)I_\alpha(y^\mu, z^\mu) - \phi^*(z^\mu)(\Gamma_{\mu\alpha}\phi)(x^\mu, z^\mu))I_\alpha(y^\mu, z^\mu) + \\ + \ g^{\mu\nu}(z^\mu)(\Gamma_{\mu\alpha}\phi^*)(x^\mu, z^\mu)(\Gamma_{\nu\alpha}\phi)(y^\mu, z^\mu) + m^2\phi^2(k)I_\alpha(x^\mu, z^\mu)I_\alpha(y^\mu, z^\mu) \ . \end{aligned} \tag{63}$$

Here, in the expressions $\Gamma_{\mu\alpha}$, $\Gamma_{\nu\alpha}$ and $\Gamma_{\rho\alpha}$, the index α should *not* be confused with μ, ν and ρ. While μ, ν and ρ are simply Lorentzian indexes, the index α is *not*. Instead, α represents a "very large" number used in $e^{-\alpha x^2/2}$. Thus, $\Gamma_{\mu\alpha} \approx \partial_\mu$ and α represents the "degree of approximation": the larger is α, the better is an approximation.

Another thing one should notice is that the definitions of $\Gamma_{0\alpha}$ and $\Gamma_{p\alpha}$ are radically different and, therefore, Lorentzian symmetry is violated:

$$(\Gamma_{0\alpha}\phi)(x^\mu, z^\mu) = \frac{\alpha^{1/2}}{(2\pi)^{1/2}\delta t}\, e^{-\frac{\alpha}{2}|\vec{x}_i - \vec{x}_k|^2}(\phi(z^\mu) - \phi(x^\mu)) \tag{64}$$

$$(\Gamma_{p\alpha}\phi)(k, i) = \frac{\alpha^{(d+2)/2}}{(2\pi)^{d/2}}\, e^{-\frac{\alpha}{2}|\vec{x}_i - \vec{x}_k|^2}(z^p - x^p)(\phi(z^\mu) - \phi(x^\mu)) \ . \tag{65}$$

Nevertheless, it is still possible to *formally* impose Einstein's summation convention

$$V^\mu \Gamma_{\mu\alpha}(x^\mu, y^\mu) = V^0\Gamma_{0\alpha}(x^\mu, y^\mu) + \sum_{p=1}^{d} V^p \Gamma_{p\alpha}(x^\mu, y^\mu) \ , \tag{66}$$

which is what we have done. Furthermore, we will postulate that the only "allowed" Lagrangian generators are the ones that can be "written down" in terms of the above summation convention. One can argue that this "restriction" upon Lagrangians is the reason why relativity appears in the lab. At the same time, the nature is fundamentally non-relativistic, as evidenced by the definitions of $\Gamma_{0\alpha}$ and $\Gamma_{p\alpha}$. One can also argue that the "continuum" version of the statement that Lorentzian indices should be contracted is also a "restriction upon Lagrangian" as opposed to a "geometrical fact". If viewed in this way, one can claim that the violation of relativity on a "geometry" level is present even in the continuum case; it is simply "hidden" by the "physics".

5. "Classical" definition of "quantum mechanical" path integral

As we recall, in Secs. 2 and 3 we have found a "classical" mechanism for *non-evolving* quantum "situation". Then, in Sec. 4 we have specified the "quantum" evolution we are "looking for" *without* proposing any "classical" mechanism. Finally, it is time to propose a "classical" mechanism that would "enforce" Sec. 4. Vaguely speaking we would like to put some "additional terms" to Secs. 2 and 3 so that, *apart from* accomplishing goals of "static" Secs. 2 and 3 we also generate a "dynamical" Sec. 4 in a "classical" manner. This corresponds to identifying $G(\cdots)$ in Eq. 11.

As we have seen from the previous section, the Lagrangian generator is a three-point function. At the same time, in order to preserve locality, we would like to find out its value by "sitting" on just one dot. Thankfully, we have already encountered similar issues in Sec. 2. The way we have dealt with these issues is by postulating that dots emit "pulses" from time to time and, whenever a given pulse is emitted, that pulse conveys the information about relevant parameters of that dot. We will now do the same exact thing, except that this time we need to know more parameters (namely, A^μ and ϕ) which we will "plug into" Lagrangian generator. Nevertheless the fact that we need more parameters simply means that we will make more and more "carbon copies" of the equations we have already proposed in Sec. 2. Thus, we have a "larger" but similar list of "messenger fields" obeying

$$\begin{aligned}
\nabla_s^\alpha \nabla_{s;\alpha} \mu_\psi + m_{\mu_\psi}^2 \mu_\psi + \lambda_\mu V^\alpha \partial_\alpha \mu_\psi &= \sum_{i=1}^{N} \psi(j) \delta^3(\vec{x} - \vec{x}_j)\, T(e_j| - \chi < e_j < \chi) \\
\nabla_s^\alpha \nabla_{s;\alpha} \mu_\phi + m_{\mu_\phi}^2 \mu_\phi + \lambda_\mu V^\alpha \partial_\alpha \mu_\phi &= \sum_{i=1}^{N} \phi_j \delta^3(\vec{x} - \vec{x}_j)\, T(e_j| - \chi < e_j < \chi) \qquad (67) \\
\nabla_s^\alpha \nabla_{s;\alpha} \mu_A^\mu + m_A^2 \mu_A^\mu + \lambda_\mu V^\alpha \partial_\alpha \mu_A^\mu &= \sum_{i=1}^{N} A^\mu(j) \delta^3(\vec{x} - \vec{x}_j)\, T(e_j| - \chi < e_j < \chi) \\
\nabla_s^\alpha \nabla_{s;\alpha} \mu_q + m_{\mu_q}^2 \mu_q + \lambda_\mu V^\alpha \partial_\alpha \mu_q &= \sum_{i=1}^{N} q_j \delta^3(\vec{x} - \vec{x}_j)\, T(e_j| - \chi < e_j < \chi) \\
\nabla_s^\alpha \nabla_{s;\alpha} \mu_{q'} + m_{\mu_q}^2 \mu_{q'} + \lambda_\mu V^\alpha \partial_\alpha \mu_q &= \sum_{i=1}^{N} q'_j \delta^3(\vec{x} - \vec{x}_i)\, T(e_j| - \chi < e_j < \chi)\ ,
\end{aligned}$$

where, as explained in Secs. 2 and 3, both the ms and the λs are too small to be felt in the interior of a given lattice but at the same time they block communication across different lattices. And, by the same argument as in Sec. 2, we will cleverly define the "functions" of these fields in such a way that they reflect the internal parameters of the dot that has emitted these fields:

$$i \text{ emits pulse } \implies A_i^\mu(\tau) \approx I_A^\mu(x^\alpha) = \mu_A^\mu(x^\alpha) |\vec{X}(x^\alpha)| = \frac{(\mu_q \mu_A^\mu)(x^\alpha)}{|\vec{\nabla} \mu_q|} \qquad (68)$$

$$i \text{ emits pulse } \implies \phi_i(\tau) \approx I_\phi(x^\alpha) = \mu_\phi(x^\alpha) |\vec{X}(x^\alpha)| = \frac{(\mu_q \mu_\phi)(x^\alpha)}{|\vec{\nabla} \mu_q|}\ . \qquad (69)$$

What we have shown so far allows us to consider "doublets"; namely, a dot we are "sitting on" (j), as well as the dot that we are "receiving" a pulse "from" (i). However, the Lagrangian generators in the previous section are *three* point functions. In order to have a *three* point function, dot number j has to "remember" exactly *two* pulses that it has received "earlier". Thus, each dot has an information about itself and, in addition to this, it also has an information about two other dots, which adds to three dots. Whenever a new pulse arrives, a dot "selects" one of these two spots into which it will store the information regarding the dot that emitted that new pulse. The information previously stored in the "selected" pulse will be erased, and replaced with the new information. On the other hand, the information previously stored in the non-selected spot will be retained in its original form.

The dot will decide which of the "spots" to "select" based on the timing of the newly arrived pulse. As usual, the dot number k will "know" the "timing" based on its "clock" $e_k(\tau)$: if $e_k(\tau) > 0$ then the information will be stored in box 1, and if $e_k(\tau) < 0$ then it will be stored in box 2:

$$(q_{j1}, \vec{x}_{j1}, \phi_{j1}, A^\mu_{j1}, \psi_{j1})(\tau + \chi) \approx$$

$$\approx \begin{cases} (I_q, \vec{I}_x, I_\phi, I^\mu_A, I_\psi)(\vec{x}_j, \tau - \chi) & \text{if } I_q(\gamma^\mu_j) \not\approx 0 \ , \ e_j > 0 \\ (q_{j1}, \vec{x}_{j1}, \phi_{j1}, A^\mu_{j1}, \psi_{j1})(\tau - \chi) & \text{otherwise} \end{cases} \tag{70}$$

$$(q_{j2}, \vec{x}_{j2}, \phi_{j2}, A^\mu_{j2}, \psi_{j2})(t + \chi) \approx$$

$$\approx \begin{cases} (I_q, \vec{I}_x, I_\phi, I^\mu_A, I_\psi)(\vec{x}_j, \tau - \chi) & \text{if } I_q(\gamma^\mu_j) \not\approx 0 \ , \ e_j > 0 \\ (q_{j1}, \vec{x}_{j1}, \phi_{j1}, A^\mu_{j1}, \psi_{j1})(\tau - \chi) & \text{otherwise} \end{cases} \tag{71}$$

The "recording" is done only during the passage of the pulse, and 2χ, as before, represents duration of the pulse. Due to the fact that $\chi \ll (N\nu)^{-1}$, we have nearly-one probability that there would be no overlap between the passage of a pulse and a transition from $e_k(\tau) < 0$ to $e_k(\tau) > 0$ or back. Thus, we know without ambiguities which of the "boxes" to record information into (or, in other words, which of the above two equations "takes place").

Now, the above "desired changes" are "enforced" for indexes 1 through the following differential equations:

$$\frac{dq_{j1}}{d\tau} = B(I_q(\gamma^\alpha_j(\tau)) - q_{j1}) e^{A(I_q(\gamma^\alpha_j(\tau)) - q_{j1}(\tau))^2} T_\epsilon \left(e_j \in \left[0, \sqrt{e_j^2 + \left(\frac{de_j}{d\tau} \right)^2} \right] \right) \tag{72}$$

$$\frac{d\vec{x}_{j1}}{d\tau} = B(\vec{I}_x(\gamma^\alpha_j(\tau)) - \vec{x}_{j1}) e^{A|\vec{I}_x(\gamma^\alpha_j(\tau)) - \vec{x}_{j1}(\tau)|^2} T_\epsilon \left(e_j \in \left[0, \sqrt{e_j^2 + \left(\frac{de_j}{d\tau} \right)^2} \right] \right) \tag{73}$$

$$\frac{d\phi_{j1}}{d\tau} = B(I_\phi(\gamma^\alpha_j(\tau)) - \phi_{j1}) e^{A|\phi(\gamma^\alpha_j(\tau)) - \phi_{j1}(\tau)|^2} T_\epsilon \left(e_j \in \left[0, \sqrt{e_j^2 + \left(\frac{de_j}{d\tau} \right)^2} \right] \right) \tag{74}$$

$$\frac{d\psi_{j1}}{d\tau} = B(I_\psi(\gamma^\alpha_j(\tau)) - \psi_{j1}) e^{A|\psi(\gamma^\alpha_j(\tau)) - \psi_{j1}(\tau)|^2} T_\epsilon \left(e_j \in \left[0, \sqrt{e_j^2 + \left(\frac{de_j}{d\tau} \right)^2} \right] \right) \tag{75}$$

The equations with index 2 are completely analogous except for the fact that $e_{j2} > 0$ is replaced with $e_{j2} < 0$. Thus,

$$\frac{dq_{j2}}{d\tau} = B(I_q(\gamma_j^\alpha(\tau)) - q_{j2})e^{A(I_q(\gamma_j^\alpha(\tau))-q_{j2}(\tau))^2}T_\epsilon\left(e_j \in \left[-\sqrt{e_j^2 + \left(\frac{de_j}{d\tau}\right)^2}, 0\right]\right) \tag{76}$$

$$\frac{d\vec{x}_{j2}}{d\tau} = B(\vec{I}_x(\gamma_j^\alpha(\tau)) - \vec{x}_{j2})e^{A|\vec{I}_x(\gamma_j^\alpha(\tau))-\vec{x}_{j2}(\tau)|^2}T_\epsilon\left(e_j \in \left[-\sqrt{e_j^2 + \left(\frac{de_j}{d\tau}\right)^2}, 0\right]\right) \tag{77}$$

$$\frac{d\phi_{j2}}{d\tau} = B(I_\phi(\gamma_j^\alpha(\tau)) - \phi_{j2}(\tau))e^{A|I_\phi(\gamma_j^\alpha(\tau))-\phi_{j2}(t)|^2}T_\epsilon\left(e_j \in \left[-\sqrt{e_j^2 + \left(\frac{de_j}{d\tau}\right)^2}, 0\right]\right) \tag{78}$$

$$\frac{d\psi_{j2}}{d\tau} = B(I_\psi(\gamma_j^\alpha(\tau)) - \psi_{j2})e^{A|I_\psi(\gamma_j^\alpha(\tau))-\psi_{j2}(\tau)|^2}T_\epsilon\left(e_j \in \left[-\sqrt{e_j^2 + \left(\frac{de_j}{d\tau}\right)^2}, 0\right]\right) \tag{79}$$

Now, in the previous chapter we have seen that the action is the integral (aka sum) of Lagrangian generators:

$$S = \sum \epsilon\mathcal{K} \, . \tag{80}$$

Therefore, the exponent of the action is the "product" of the exponents:

$$e^{-iS} = \prod e^{-i\xi\mathcal{K}} = \prod(1 - i\xi\mathcal{K}) \, . \tag{81}$$

Unlike everything else that we have done, the "changes" related to the above "product" are best to me made during the interval *between* the pulses. After all, we would like to "know" what $\mathcal{K}$ is in order to use it. During the passage of the signal we "don't know" $\mathcal{K}$: after all, its arguments are "no longer" equal to their "former" values, but they have not achieved their new values "yet". On the other hand, in the time interval between the pulses both the relevant parameters, as well as $\mathcal{K}$ itself, are well defined. Thus, we use that interval to "evolve" ψ_j:

$$\text{No Signal} \implies \frac{d\psi_j}{d\tau} = -i\xi T[q_{j1}(\tau) \approx q_{j2}(\tau)]\mathcal{K}(\vec{x}_{j1}, \phi_{j1}, A^\mu_{j1}, \psi_{j1}; \vec{x}_{j2}, \phi_{j2}, A^\mu_{j2}, \psi_{j2}) \, . \tag{82}$$

It is important to notice that the above only "updates" the value of ψ attached to dot j; but when we are talking of a probability amplitude we are referring to the one associated with *all dots* having "the same tuning" as j. The reason we were only focused on j is that we know, from Sec. 3, that whatever happens to the probability amplitude at j ends up "spreading out" elsewhere. Of course, in many cases the information will "get lost" if the "first" dot emitting a given signal happens to be something "other than" j. But there are "few" cases when j is "lucky enough" to be the "first" one to emit signal in which case it passes down its change in its entirety. On average, the change simply gets divided by relevant number of dots, which simply means "slowing down" the evolution.

Now, the values of q_{j1} and q_{j2} are *not* integers. After all, they are supposed to "record" the values of q which are only *approximations* to integers. Furthermore, the "records" that are "stored" in q_{j1} and q_{j2} should be able to "change". Since the "change" occurs

in differentiable fashion, these are guaranteed to be non-integer at least during the short transition period (but, of course, these are not to be confused with q_j which *is* an integer). However, it is safe to say that they are "good approximations" to integers *most of the time*, and the transition period when approximation breaks down has negligible influence. Thus, we use our usual "1/2" trick in order to define what we mean by approximating an integer:

$$\text{No Signal } \Longrightarrow \frac{d\psi_j}{d\tau} = -i\xi T_\epsilon\Big(q_{j2}(t) \in \Big[q_{j1}(t) - \frac{1}{2}, q_{j1}(t) + \frac{1}{2}\Big]\Big)\times$$
$$\times\ \mathcal{K}(\vec{x}_{j1}, \phi_{j1}, A^\mu_{j1}, \psi_{j1}; \vec{x}_{j2}, \phi_{j2}, A^\mu_{j2}, \psi_{j2}) \tag{83}$$

As a final step, we add the dynamics of ψ_k we have proposed in Chapter 2 to the one we have proposed now, and thus obtain the final equation for ψ_j:

$$\frac{d\psi_j}{d\tau} = -i\xi T_\epsilon\Big(q_{j2}(t)|q_{j1}(t) - \frac{1}{2} < q_{j2}(t) < q_{j1}(t) + \frac{1}{2}\Big)\mathcal{K}(\vec{x}_{j1}, \phi_{j1}, A^\mu_{j1}, \psi_{j1}; \vec{x}_{j2}, \phi_{j2}, A^\mu_{j2}, \psi_{j2}) +$$
$$+ B(I_\psi(\vec{x}_j, t) - \psi_j(t))e^{A(\psi(\vec{x}_j,t)-\psi_j(t))^2} T_\epsilon\Big(I_q(\vec{x}_j, t) \in \Big[q_j - \frac{1}{2}, q_j + \frac{1}{2}\Big]\Big). \tag{84}$$

Here, the first line represents the evolution of ψ under *nearly constant* $\mathcal{K}$ *without* passing signal, while the second line represents the reaction to the signal that is meant to "equalize" ψ among dots of the same tuning.

6. Conclusion

In this paper we have presented a model of "classical" spin-0 and electromagnetic fields in the Feynman path-integral representation. We have proposed a model through which a lattice forms and, once its formed, it goes through classical processes that approximate discretized quantum field theory. Here, by "discretization" we mean both the discretization of the lattice itself as well as the discretization of the field trajectories. The latter reduces the informational context of the path integral to the point that it can be embedded in four dimensions in a "classical" form.

It should be pointed out that another paper in field-based representation was written recently [2]. In that paper, the quantum electrodynamics was likewise reproduced, again through specification of $\mathcal{K}$. In fact, we have simply borrowed the Lagrangian generator $\mathcal{K}$ from that other paper and expressed it in a more transparent form. However, we have completely changed the part that involves "signals" and made it a lot more simpler. The idea that we share with Ref. [2] is the fact that dots communicate through short pulses. However, in Ref. [2] we were encoding both probability amplitudes of states as well as transition amplitudes between states. On the other hand, in the current paper we realized that we don't need to encode the "transition amplitudes". We can use $\mathcal{K}$ directly in order to evolve "probability amplitudes". This has greatly simplified a lot of things.

At the same time, in this paper we are only doing a small portion of what was done in Ref. [2]. In particular, Ref. [2] includes fermionic fields (in Section 3.2) and it also includes the

chapter on gravity-based GRW model of measurement (Sec. 5). Neither of these two topics were done in this paper. Besides, Ref. [2] shows how these different topics are interconnected, whereas the current paper is only focused on the bosonic fields. For this reason we strongly recommend that the reader looks at Ref. [2]. But, at the same time, we are planning to make improvements/simplificaitons of other parts of Ref. [2] in the upcoming papers.

As was also said in the conclusion of Ref. [2], The availability of "classical" model opens doors to answering some of the questions that have previously been unanswered, such as for example Hawking radiation. It should be noticed that the current work was based on the assumption of "flat space QFT" (or "Peskin and Schroeder" [4], so to speak). At the same time, we can blindly "plug in" our "classical" theory into the "curved space" context. At the same time, we can make it a point *not* to do any "adjustments" for the well known curved space QFT (such as the one given in Birrell and Davies [5]). By deliberately sticking to the "classical" theory designed for Peskin and Schroeder, the agreement or disagreement with Birrell and Davies will become a non-trivial issue. In case of disagreement between our work and Birrell and Davies, we would be able to come up with experimental tests of our theory. In light of the fact that Birrell and Davies was not as widely tested as Peskin and Schroeder was, it is "conceivable" that our result will match the experiment closer; but of course we don't know one way or the other at this stage.

It might also be interesting to see what would happen if, instead of assuming that c_s is large enough for the signals to pass all of the lattice points within a "very small time", we will assume that they can only circle the galaxy but not the entire universe. In this case, the QFT will break down on a cosmological scales. This logically leads to the idea that physics on cosmological scale obey some other version of QFT, which is based on several Hilbert spaces (rather than just one) over various overlapping domains? quantum states over "smaller domains"? This might also lead us to ask whether or not this "different QFT" will make some cosmological predictions that conflict with current QFT and perhaps explain things such as dark matter. Again, at the moment the connection between this idea and dark matter problem is simply a blind guess. All we know is that QFT will be "different" but we don't know "in what way" it would be different. But at least it is something worth looking into.

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